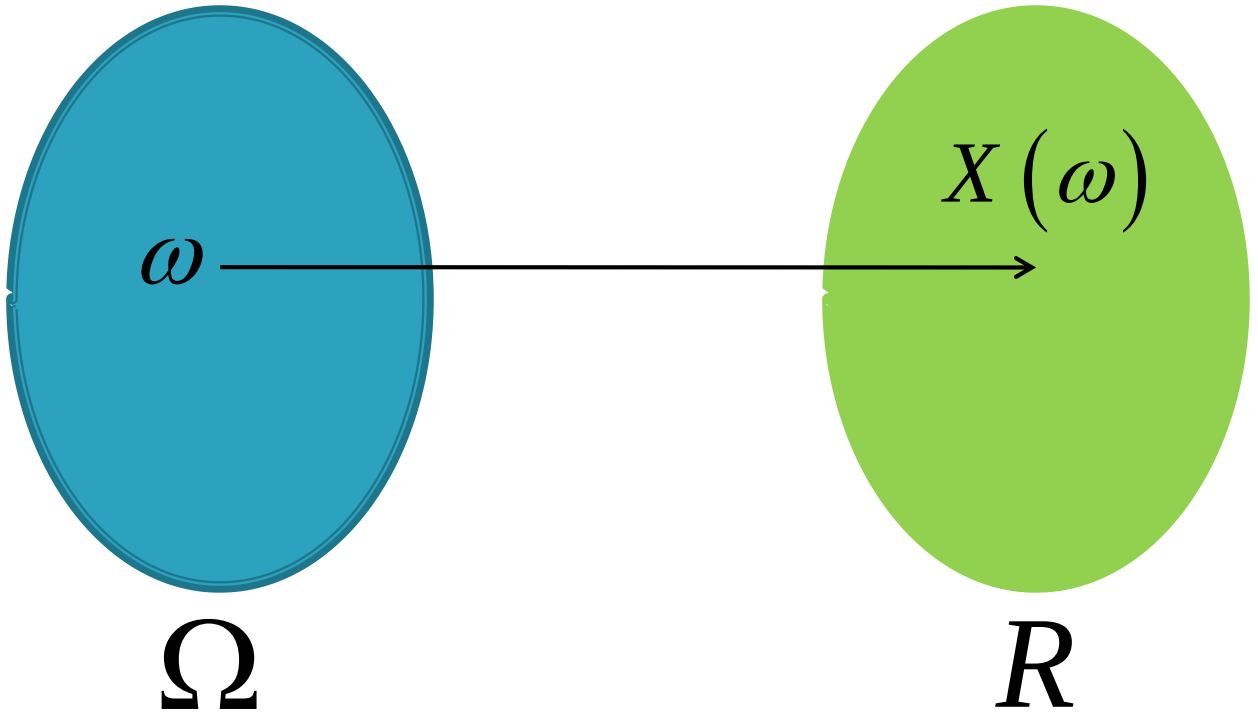
System of Equations with Random Set Parameters

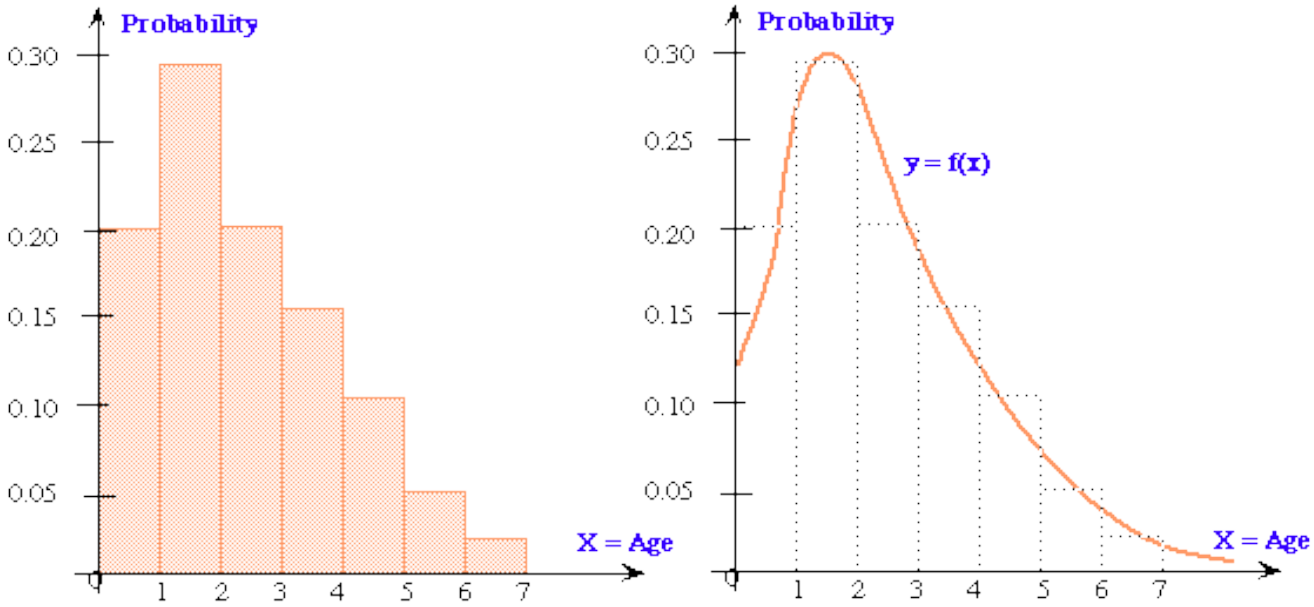
Jorge Alonso
Andrzej Pownuk

Random variable

$$X : \Omega \ni \omega \to X(\omega) \in R$$



Probability density function



Random set

$$X(\omega) = \left[\underline{X}(\omega), \overline{X}(\omega) \right]$$

$$p_i = P_\Omega \{\omega_i\}$$

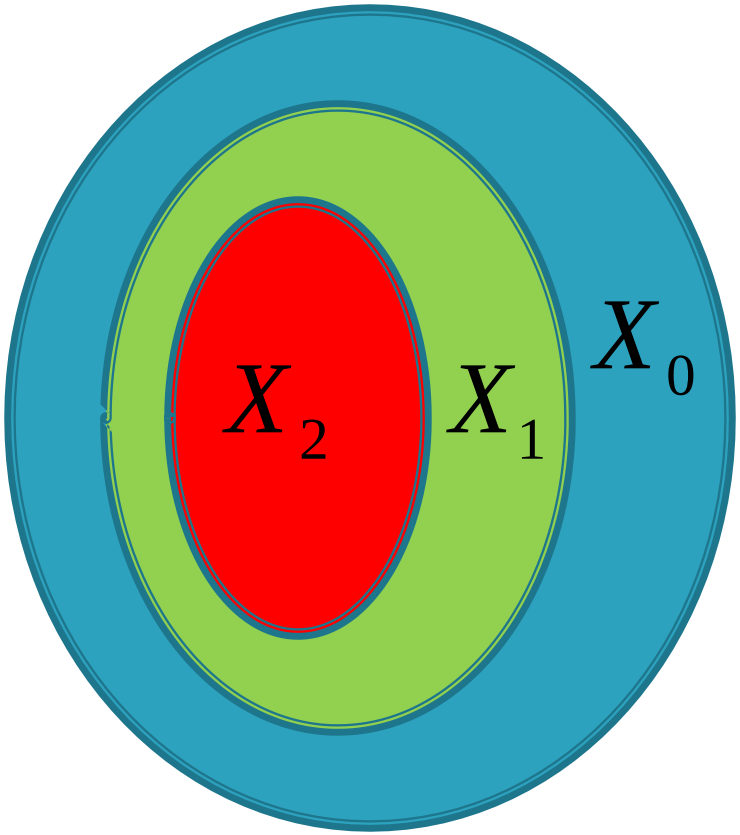
$$\sum_{\omega_i \in \Omega} P_\Omega \{\omega_i\} = \sum_i p_i = 1$$

Upper and lower probability

$$Pl(A) = \sum_{i:A \cap X_i \neq \emptyset} p_i$$

$$Bel(A) = \sum_{i:X_i \subseteq A} p_i$$

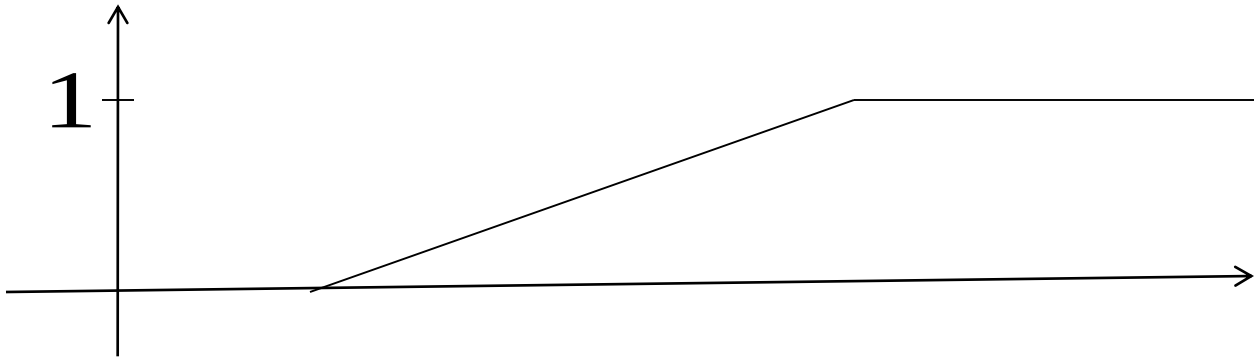
The theory of clouds



$$p_i = P_\Omega \left\{ X \in X_i \right\}$$

Cumulative distribution function

$$F(x) = P_{\Omega}\{X \leq x\}$$

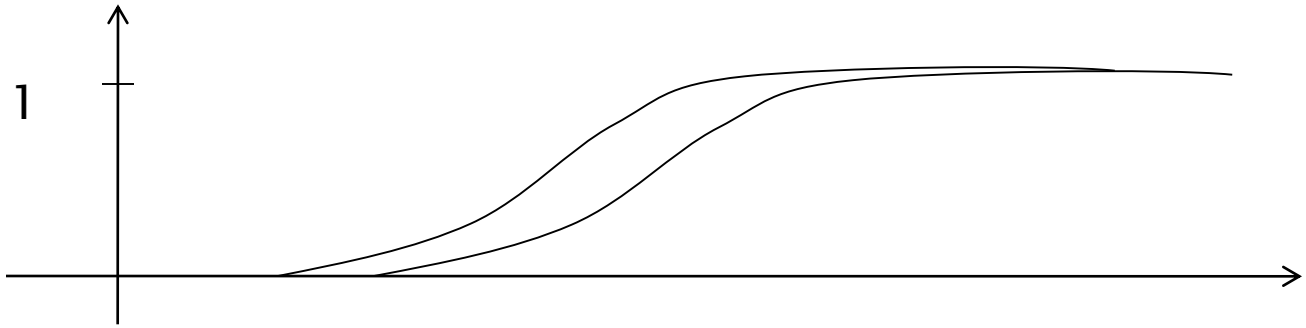


$$P([a,b]) = F(b) - F(a)$$

Cumulative distribution function in the case of random sets

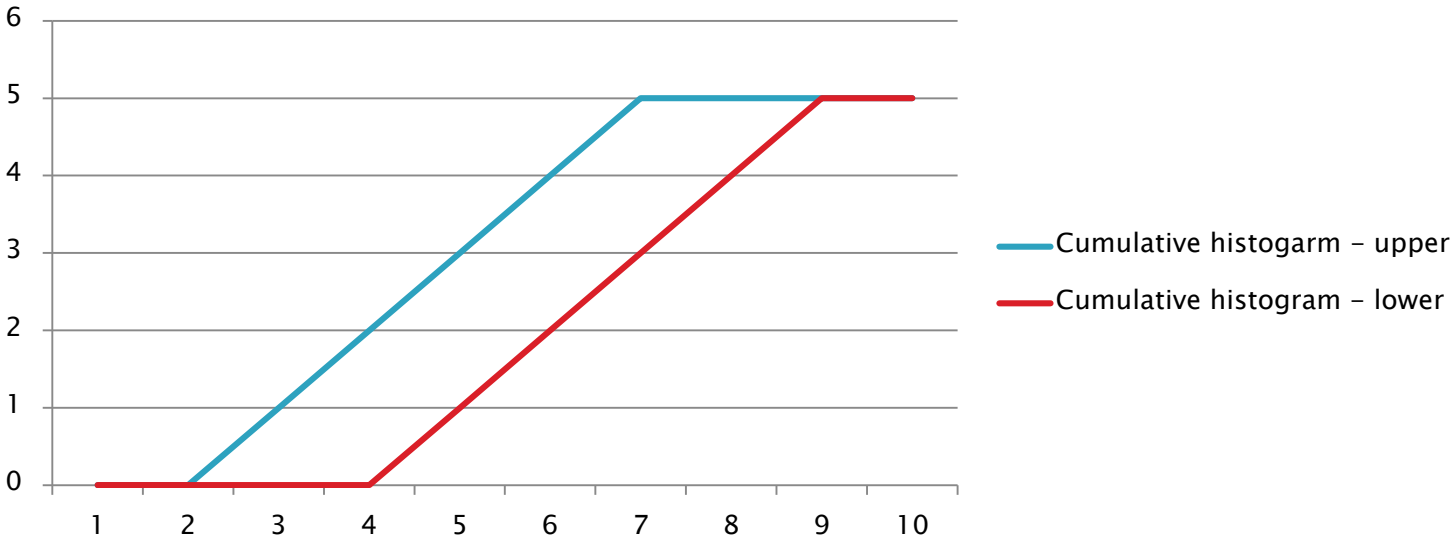
$$\underline{F}(x) = P_{\Omega}\left\{\overline{X} \leq x\right\}$$

$$\overline{F}(x) = P_{\Omega}\left\{\underline{X} \leq x\right\}$$



Example – upper probability

Upper	0	0	1	2	3	4	5	5	5	5
Lower	0	0	0	0	1	2	3	4	5	5



Upper probability

$$Pl\left([a,b]\right) = \overline{F}\left(b\right) - \underline{F}\left(a\right)$$

Functions of random variables

$$y = g(x)$$

$f_X(x)$ – PDF of the random variable *X.*

$$f_Y(y) = f_X\left(g^{-1}(y)\right)\left|\frac{dg^{-1}(y)}{dx}\right|$$

It is necessary to describe random sets by using Monte Carlo simulations.

Computational method

- 1) Generate the interval

$$\mathbf{X}_i = \left[\underline{X}_i, \overline{X}_i \right]$$

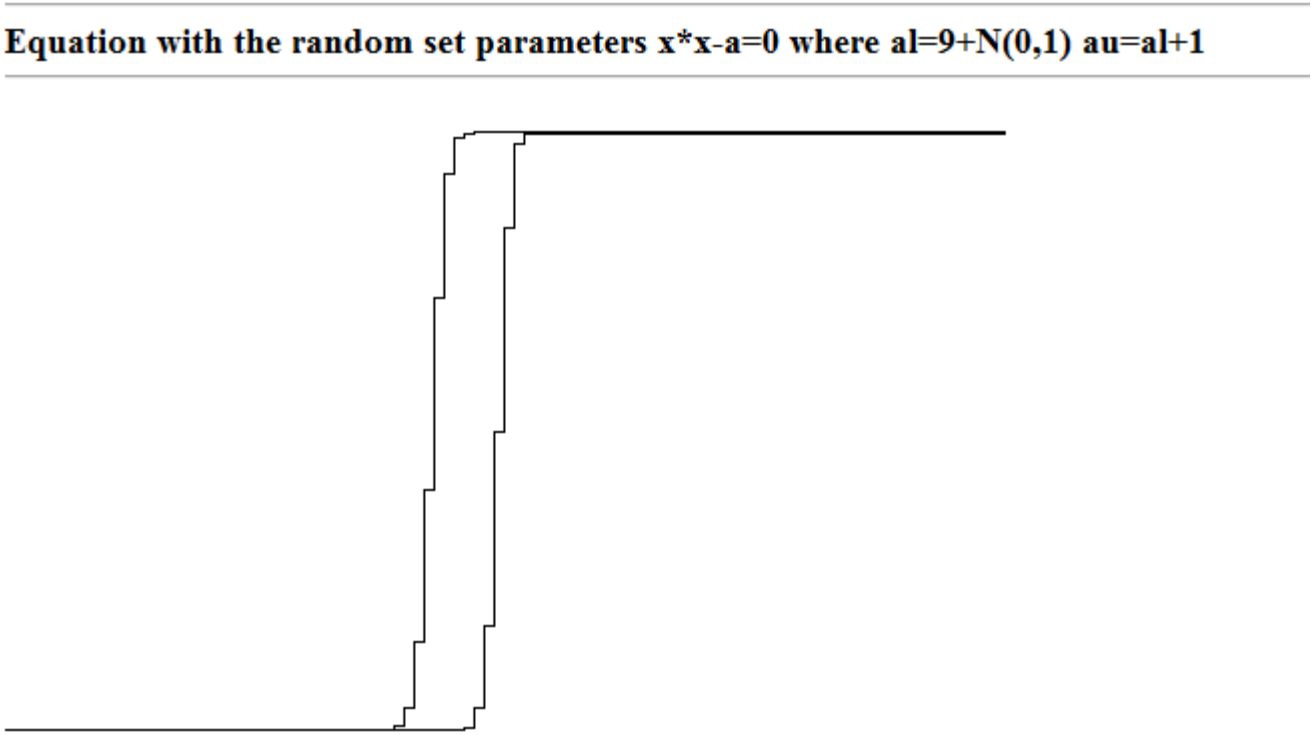
- 2) Calculate

$$\mathbf{Y}_i = \left[\underline{Y}_i, \overline{Y}_i \right] = g\left(\mathbf{X}_i \right)$$

- 3) Update $\underline{F}_i(y), \overline{F}_i(y)$.
- 4) Return to 1.

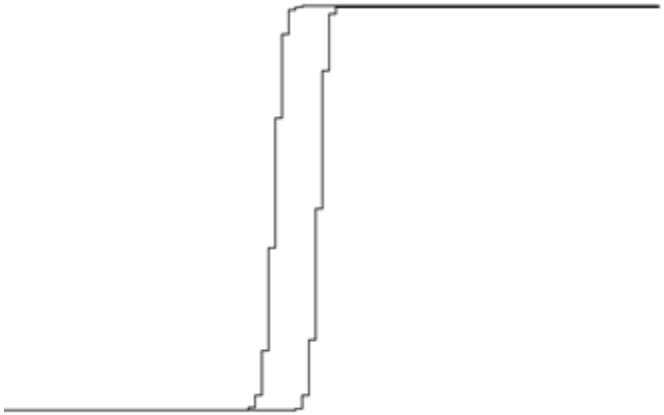
Numerical example

http://andrzej.pownuk.com/java/IntervalHistogramEquation/IntervalHistogramEquation.html



System of equations

$$\begin{cases} ax + by = a^2 + b^2 \\ ax - by = a^2 - b^2 \end{cases}$$



Conclusions

- Using special version of Monte Carlo simulations it is possible to solve system of equations with the random-sets parameters.
- Example solution are available on-line and they are implemented in special java applets.